Limites.

9a $\lim\limits_{x \to 1} \dfrac{x^3+2x^2-x-2}{x^2+x-2} = \dfrac{1+2-1-2}{1+1-2} = "\dfrac{0}{0}"$

Horner

	1	2	−1	−2
1		1	3	2
	1	3	2	0
−1		−1	−2	
	1	2	0	

$\lim\limits_{1} \dfrac{\cancel{(x-1)}.(x+1).\cancel{(x+2)}}{\cancel{(x+2)}.\cancel{(x-1)}}$

$= \lim\limits_{1} (x+1) = 2.$ Pt exclu $(1;2)$

b $\lim\limits_{x \to 5} \dfrac{x^2-3x+2}{x^2-6x+5} = \dfrac{25-15+2}{25-30+5} = "\dfrac{12}{0}"$

$x^2-6x+5 = (x-1).(x-5)$

x		1		5	
x^2-6x+5	+	0	−	0	+

$\lim\limits_{5^-} f(x) = \dfrac{12}{0^-} = -\infty$ $\Rightarrow$ A.V. $\equiv x=5$

$\lim\limits_{5^+} f(x) = \dfrac{12}{0^+} = +\infty$

c $\lim\limits_{x \to -\infty} (x^3-5x+3) = \lim\limits_{-\infty} x^3 = -\infty$

d $\lim\limits_{x \to +\infty} \dfrac{1-4x^2}{x^2+7} = "\dfrac{\infty}{\infty}"$

$\lim\limits_{+\infty} \dfrac{-4\cancel{x^2}}{\cancel{x^2}} = -4$ $\Rightarrow$ A.H. $\equiv y=-4$

e $\lim\limits_{x \to +\infty} \dfrac{3x^2-7x+1}{4x-5x^2} = "\dfrac{\infty}{\infty}"$

$\lim\limits_{+\infty} \dfrac{3\cancel{x^2}}{-5\cancel{x^2}} = -\dfrac{3}{5}$ $\Rightarrow$ A.H. $\equiv y = -\dfrac{3}{5}$

10. $f_1(x) = \frac{2x+1}{x-5} = \frac{2x-10+11}{x-5} = 2 + \frac{11}{x-5}$

$\text{Dom } f_1 = \mathbb{R} \setminus \{5\}$

A. H.

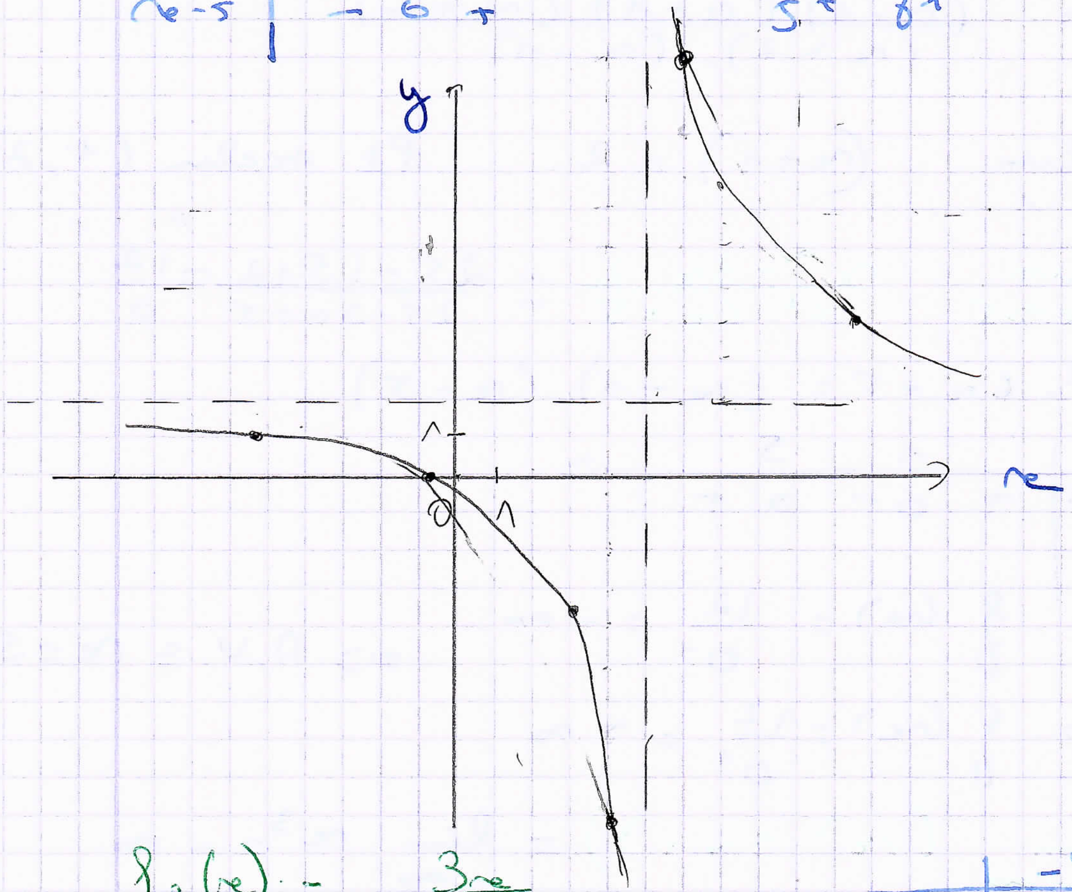
$\lim_{\pm\infty} f_1(x) = \lim_{\pm\infty} \frac{2x}{x} = 2 \Rightarrow \text{A.H.} \equiv y = 2.$

A. V.

$\lim_{5} f_1(x) = \frac{11}{0}$ $\qquad \lim_{5^-} f_1(x) = \frac{11}{0^-} = -\infty$

$\lim_{5^+} f_1(x) = \frac{11}{0^+} = +\infty.$

x		5	
$x-5$	$-$	0	$+$



$f_2(x) = \frac{3x}{16-x^2}.$

x		-4		4	
$16-x^2$	$-$	0	$+$	0	$-$

$\text{Dom } f_2 = \mathbb{R} \setminus \{-4, 4\}$

A. H.

$\lim_{\pm\infty} f_2(x) = \lim_{\pm\infty} \frac{3x}{-x^2} = 0 \Rightarrow \text{A.H.} \equiv y = 0.$

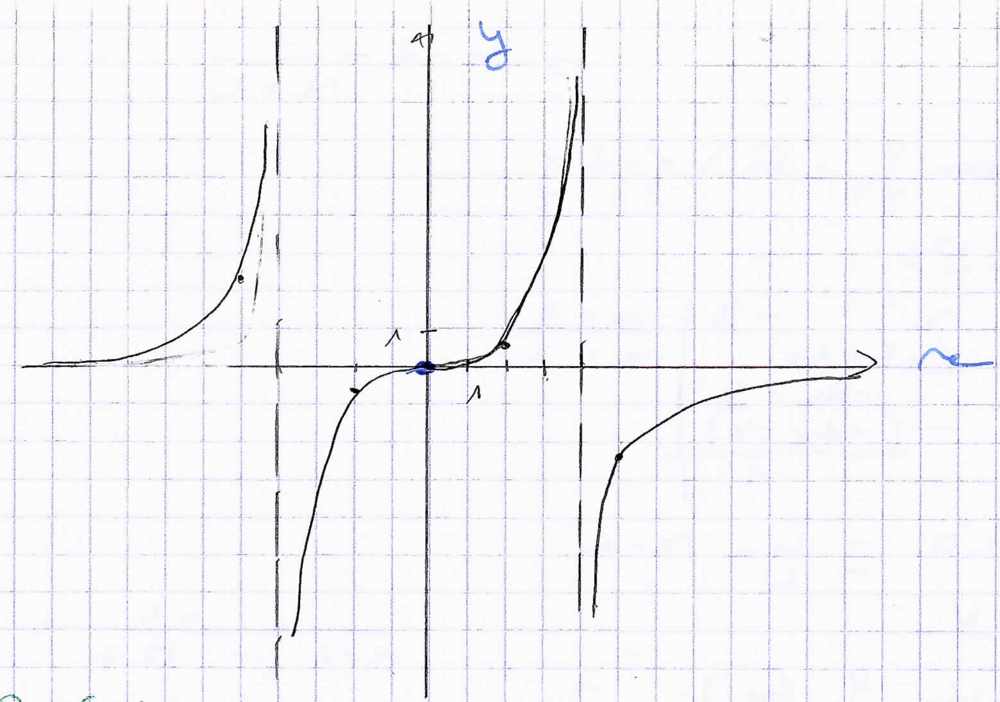
A. V.

$\lim_{4} f_2(x) = \frac{12}{0}$ $\qquad \lim_{4^-} f_2(x) = \frac{12}{0^+} = +\infty \Rightarrow \text{A.V.} \equiv x = 4$

$\lim_{4^+} f_2(x) = \frac{12}{0^-} = -\infty$

$\lim_{-4} f_2(x) = \frac{-12}{0}$ $\qquad \lim_{-4^-} f_2(x) = \frac{-12}{0^-} = +\infty \Rightarrow \text{A.V.} \equiv x = -4$

$\lim_{-4^+} f_2(x) = \frac{-12}{0^+} = -\infty$



$\frac{15}{-9} = -\frac{5}{3}$

$f_3(x) = \frac{x^2 - x - 2}{x^2 - 2x - 3}$

		-1		3	
D	$+$	0	$-$	0	$+$

Dom $f_3 = \mathbb{R} \setminus \{+3 ; -1\}$

A. H.

$\lim_{\pm\infty} f_3(x) = \lim_{\pm\infty} \frac{x^2}{x^2} = 1 \Rightarrow$ A. H. $\equiv y = 1$

A. V.

- $\lim_{3} f_3(x) = \frac{9-3-2}{0} = \text{"}\frac{4}{0}\text{"}$

$\lim_{3^-} f_3(x) = \frac{4}{0^-} = -\infty$ $\Rightarrow$ A. V. $\equiv x = 3$

$\lim_{3^+} f_3(x) = \frac{4}{0^+} = +\infty$

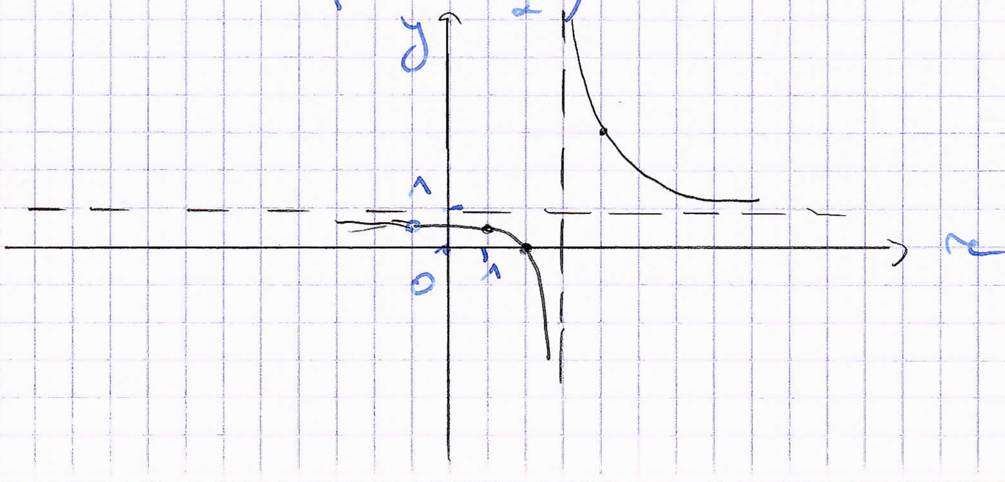
- $\lim_{-1} f_3(x) = \frac{1+1-2}{1+2-3} = \text{"}\frac{0}{0}\text{"}$

$\lim_{-1} \frac{(x+1).(x-2)}{(x+1)(x-3)} = \lim_{-1} \frac{x-2}{x-3} = \frac{-1}{-2} = \frac{1}{2}$

Pt exclu $\left(-1 ; \frac{1}{2}\right)$

$\frac{-2}{-4} = \frac{1}{2}$

$\frac{10}{5} = 2$



$f_4(x) = \dfrac{x^2-3}{x+2} = x - 2 + \dfrac{1}{x+2}$

$\text{Dom } f_4 = \mathbb{R} \setminus \{-2\}$

A. O.

$$\begin{array}{r|l} x^2 \quad\quad -3 & x+2 \\ -(x^2+2x) \quad\quad & x-2 \\ -2x-3 & \\ -(-2x-4) & \\ 1 & \end{array}$$

A. O. $\equiv y = x-2.$

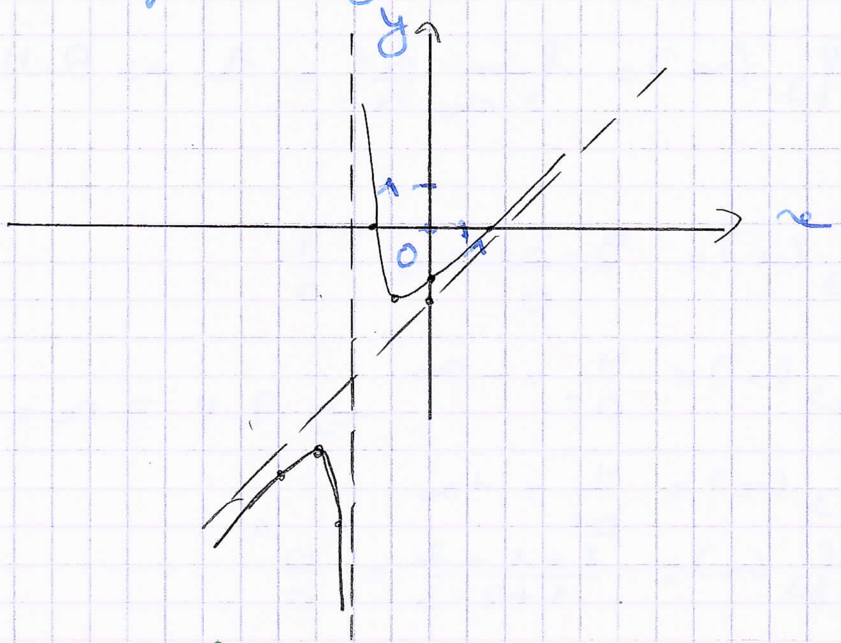
A. V

$\lim\limits_{-2} f_4(x) = \text{" } \dfrac{1}{0} \text{ "}$

x		-2	
$x+2$	$-$	0	$+$

$\lim\limits_{-2^-} f_4(x) = \dfrac{1}{0^-} = -\infty$

$\lim\limits_{-2^+} f_4(x) = \dfrac{1}{0^+} = +\infty$ $\Rightarrow$ A. V. $\equiv x = -2.$



$-1 \quad -\dfrac{2}{1}$

$-3 \quad \dfrac{6}{-1}$

$f_5(x) = x^3 - 2x^2 + x - 5.$

$\text{Dom } f_5 = \mathbb{R}$. f ct polynomiale donc pas d'asymptotes.

$$f_6(x) = \frac{3x^2+1}{x^2+x-6}$$

		-3		2	
D	$+$	0	$-$	0	$+$

$\text{Dom } f_6 = \mathbb{R} \setminus \{-3 ; 2\}$

A.H.

$\lim\limits_{\pm\infty} f_6(x) = \lim\limits_{\pm\infty} \frac{3x^2}{x^2} = 3 \Rightarrow \text{A.H.} \equiv y = 3$

A.V.

- $\lim\limits_{-3} f_6(x) = \text{"}\frac{28}{0}\text{"}$

$\lim\limits_{-3^-} f_6(x) = \frac{28}{0^+} = +\infty$

$\lim\limits_{-3^+} f_6(x) = \frac{28}{0^-} = -\infty$ $\Rightarrow \text{A.V.} \equiv x = -3$

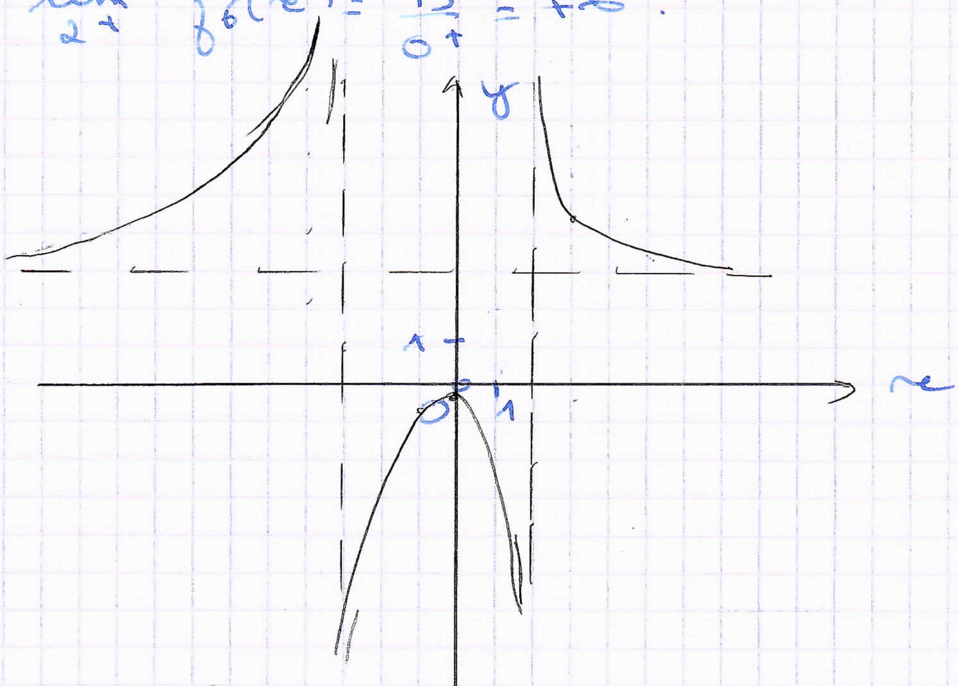
- $\lim\limits_{2} f_6(x) = \text{"}\frac{13}{0}\text{"}$

$\lim\limits_{2^-} f_6(x) = \frac{13}{0^-} = -\infty$

$\lim\limits_{2^+} f_6(x) = \frac{13}{0^+} = +\infty$. $\Rightarrow \text{A.V.} \equiv x = 2$.

$\frac{28}{6} = 4,$

$\frac{49}{6} = 8,$



$$f_7(x) = \frac{2x^2+3x-1}{-x-2} = -2x+1 + \frac{1}{-x-2}$$

$\text{Dom } f_7 = \mathbb{R} \setminus \{-2\}$

A.O.

$$\begin{array}{r|l} 2x^2+3x-1 & -x-2 \\ \hline -(2x^2+4x) & -2x+1 \\ -x-1 & \\ -(-x-2) & \\ \hline 1 & \end{array}$$

$\text{A.O.} \equiv y = -2x+1$.

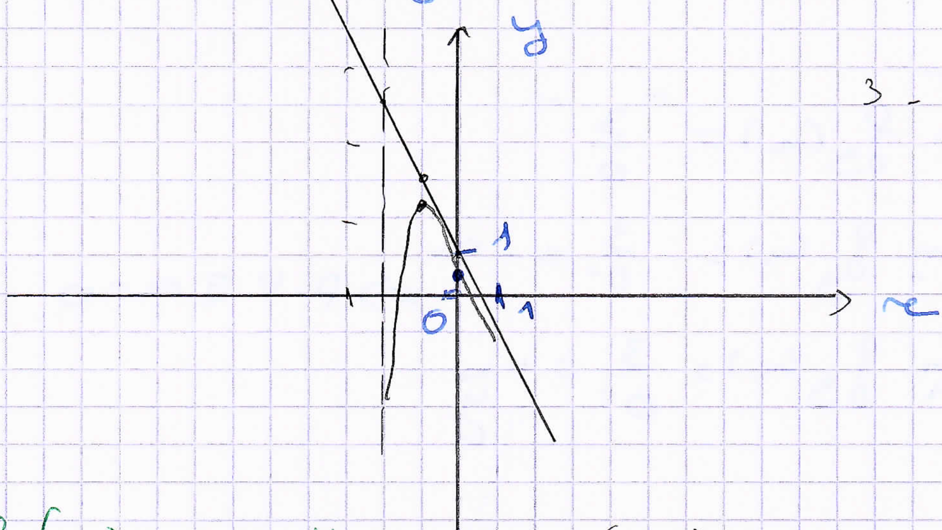
A.V.

$\lim\limits_{-2} f_7(x) = \text{"}\frac{1}{0}\text{"}$

$-x-2$ | $+ \quad 0 \quad -$

$\lim\limits_{-2^-} f_7(x) = \frac{1}{0^+} = +\infty$ $\Rightarrow$ A.V. $\equiv x = -2$.

$\lim\limits_{-2^+} f_7(x) = \frac{1}{0^-} = -\infty$.



$3 - \frac{1}{2}$

$f_8(x) = \frac{x-4}{x^2-6x+9} = \frac{(x-4)}{(x-3)^2}$

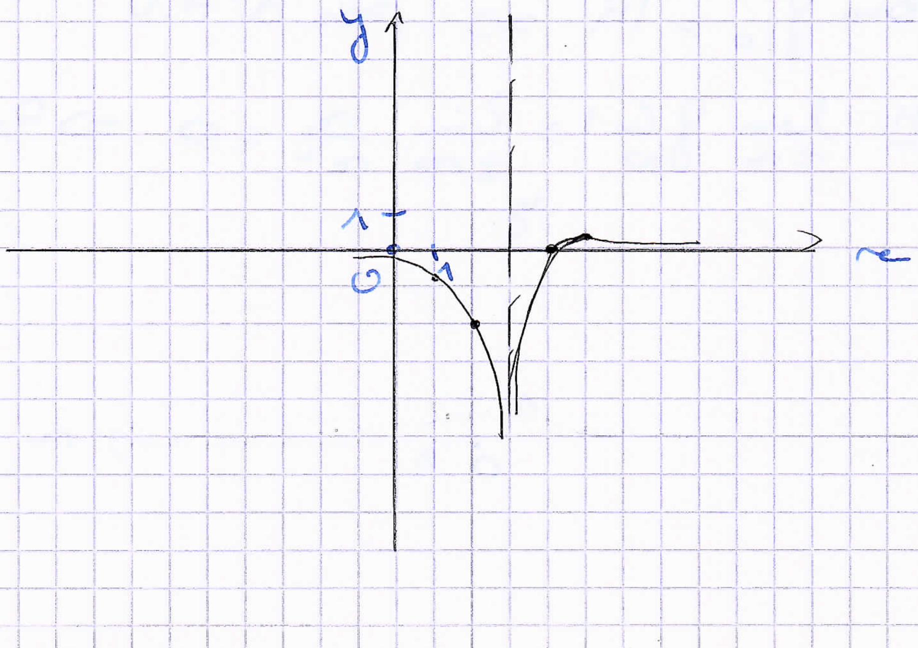
$\text{Dom } f_8 = \mathbb{R} \setminus \{3\}$

A.H

$\lim\limits_{\pm\infty} f_8(x) = \lim\limits_{\pm\infty} \frac{x}{x^2} = 0$ $\Rightarrow$ A.H $\equiv y = 0$

A.V

$\lim\limits_{3} f_8(x) = \frac{-1}{0^+} = -\infty$. $\Rightarrow$ A.V. $\equiv x = 3$.



$-\frac{3}{4}$

-2

$f_9(x) = \frac{3-x}{2x}$

Dom $f_9 = \mathbb{R}_0$

A.H

$\lim_{\pm\infty} f_9(x) = \lim_{\pm\infty} -\frac{x}{2x} = -\frac{1}{2} \Rightarrow$ A.H. $\equiv y = -\frac{1}{2}$

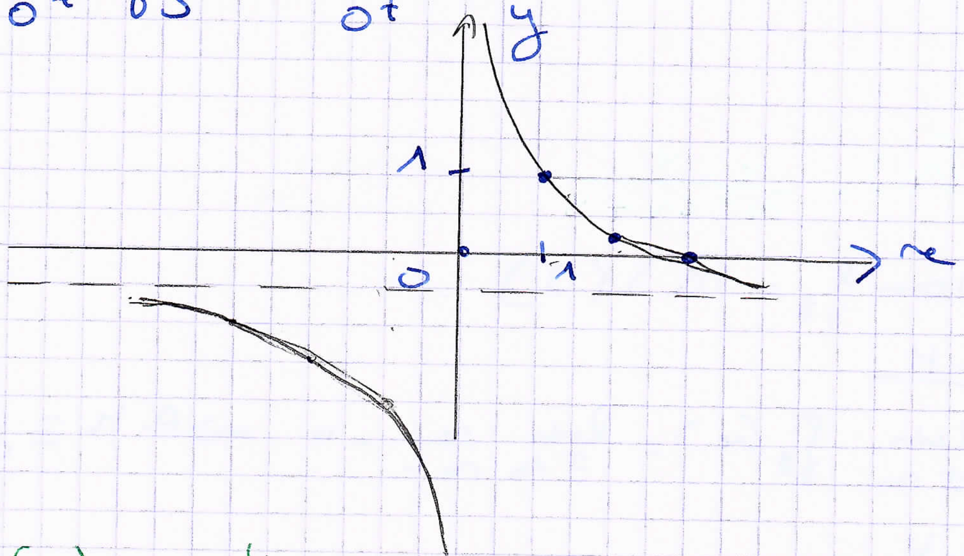
A.V

$\lim_{0} f_9(x) = "\frac{3}{0}"$

$\lim_{0^-} f_9(x) = \frac{3}{0^-} = -\infty$

$\Rightarrow$ A.V. $\equiv x = 0$.

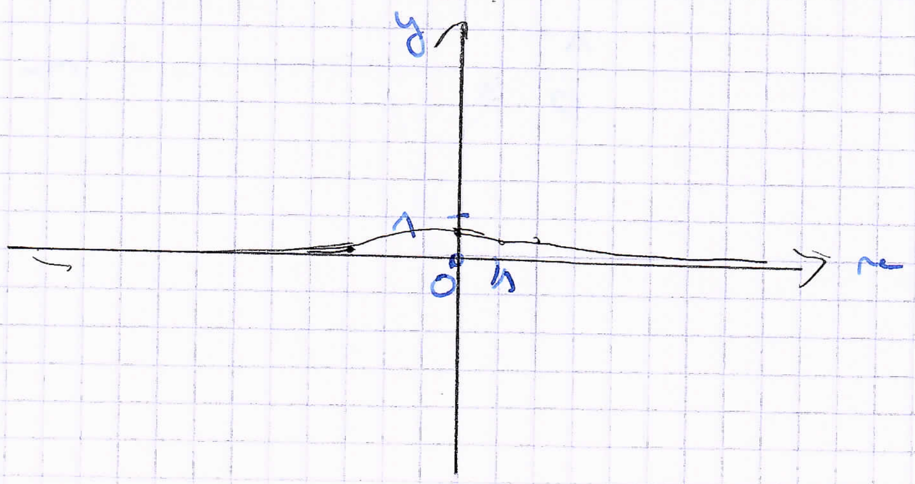
$\lim_{0^+} f_9(x) = \frac{3}{0^+} = +\infty$



$f_{10}(x) = \frac{4+x}{x^2+2x+8}$

Dom $f_{10} = \mathbb{R} \Rightarrow$ pas d'A.V.

A.H. $\lim_{\pm\infty} f_{10}(x) = \lim_{\pm\infty} \frac{x}{x^2} = 0 \Rightarrow$ A.H. $\equiv y = 0$.



$f_{11}(x) = \dfrac{x^3+3x^2-4x-12}{x^2-9} = x+3 + \dfrac{32x+15}{x^2-9}$

$\text{Dom } f_{11} = \mathbb{R} \setminus \{-3\,;\,3\}$

A.O.

$$\begin{array}{l|l} x^3+3x^2-4x-12 & x^2-9 \\ -(x^3 \quad\quad -9x) & x+3 \\ \hline 3x^2+5x-12 & \\ -(3x^2 \quad\quad -27) & \\ \hline 32x+15 & \end{array}$$

$\Rightarrow$ A.O. $\equiv y = x+3$.

$$\begin{array}{c|ccccc} x^2-9 & & -3 & & 3 & \\ & + & 0 & - & 0 & + \end{array}$$

A.V.

• $\lim\limits_{3} f_{11}(x) = \dfrac{27+27-12-12}{0} = \text{"}\dfrac{30}{0}\text{"}$

$\lim\limits_{3^-} f_{11}(x) = \dfrac{30}{0^-} = -\infty$

$\lim\limits_{3^+} f_{11}(x) = \dfrac{30}{0^+} = +\infty$ $\Rightarrow$ A.V. $\equiv x = 3$

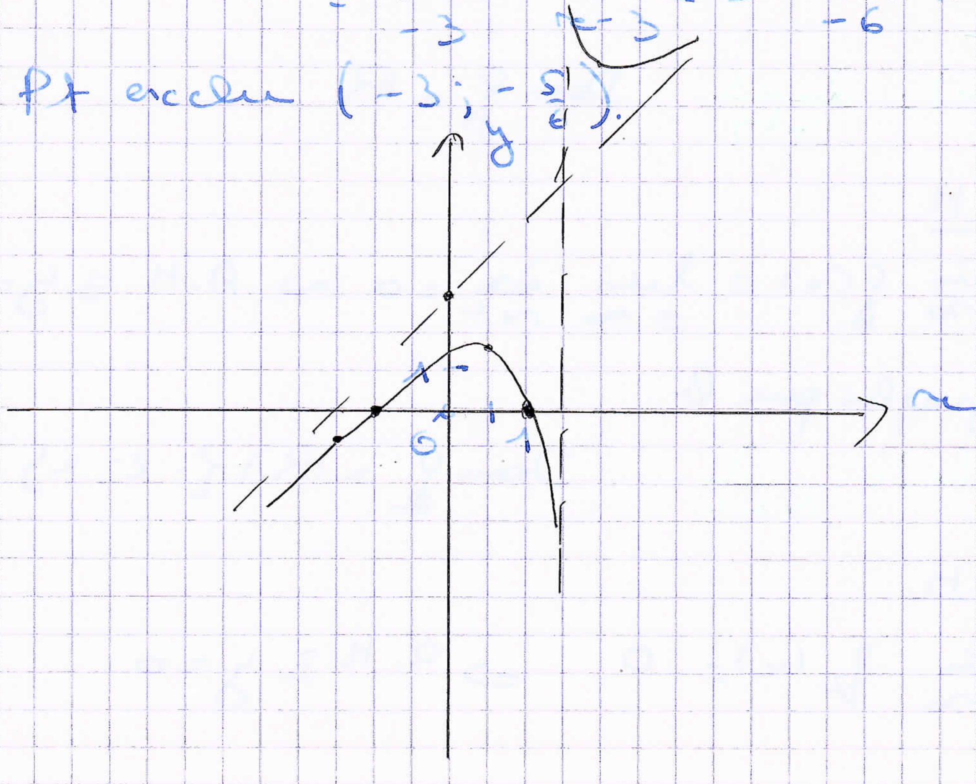
• $\lim\limits_{-3} f_{11}(x) = \dfrac{27-27+12-12}{0} = \text{"}\dfrac{0}{0}\text{"}$

$$\begin{array}{c|ccc|c} & 1 & 3 & -4 & -12 \\ -3 & & -3 & 0 & 12 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$\lim\limits_{-3} f_{11}(x) = \lim\limits_{-3} \dfrac{(x+3)(x^2-4)}{(x+3)(x-3)}$

$= \lim\limits_{-3} \dfrac{x^2-4}{x-3} = \dfrac{5}{-6} = -\dfrac{5}{6}$

Pt exclu $(-3\,;\,-\frac{5}{6})$.



$f_{12}(x) = \dfrac{2x^2 - 5x + 3}{2-x} = -2x + 1 + \dfrac{1}{2-x}$

Dom $f_{12} = \mathbb{R} \setminus \{2\}$

A.O.

$$\begin{array}{r|l} 2x^2 - 5x + 3 & 2-x \\ -(2x^2 - 4x) & \overline{-2x+1} \\ \hline -x + 3 & \\ -(-x + 2) & \\ \hline 1 & \end{array}$$

$\Rightarrow$ A.O. $\equiv y = -2x + 1$

x		2	
$2-x$	$+$	0	$-$

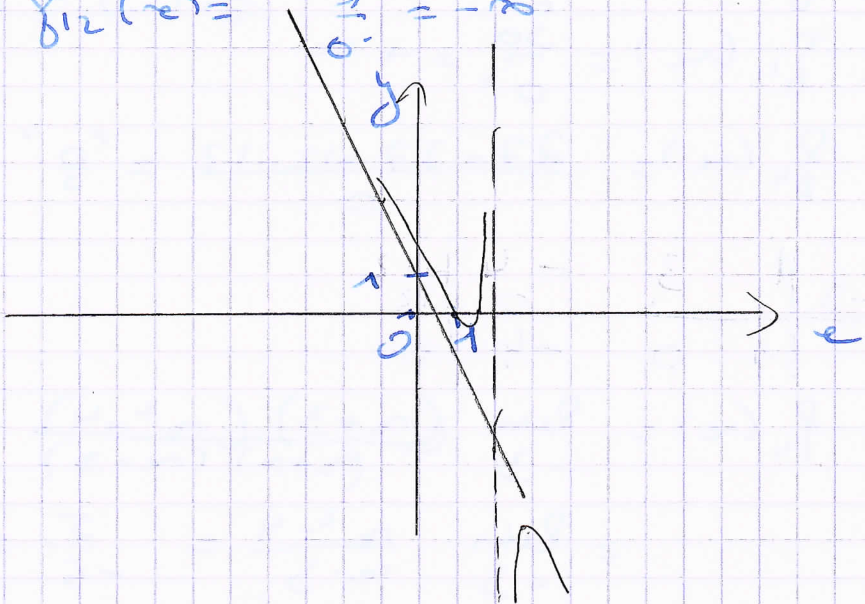
A.V.

$\lim\limits_{2} f_{12}(x) = \dfrac{2 \cdot 4 - 10 + 3}{0} = "\dfrac{1}{0}"$

$\lim\limits_{2^-} f_{12}(x) = \dfrac{1}{0^+} = +\infty$

$\lim\limits_{2^+} f_{12}(x) = \dfrac{1}{0^-} = -\infty$

$\Rightarrow$ A.V. $\equiv x = 2$.



11

$f_1(x) = \dfrac{2x+3}{x^2+4}$ Dom $f_1 = \mathbb{R}$

A.H.

$\lim\limits_{\pm\infty} f(x) = \lim\limits_{\pm\infty} \dfrac{2x}{x^2} = 0 \Rightarrow$ A.H $\equiv y = 0$

graphique ?

$f_2(x) = \dfrac{3}{x^2-4}$ Dom $f_2 = \mathbb{R} \setminus \{-2, 2\}$

A.H.

$\lim\limits_{\pm\infty} f_2(x) = 0 \Rightarrow$ A.H. $\equiv y = 0$

x		-2		2	
x^2-4	$+$	0	$-$	0	$+$

A.V.

• $\lim\limits_{2} f_2(x) = \text{"}\frac{3}{0}\text{"}$

$\lim\limits_{2^-} f_2(x) = \frac{3}{0^-} = -\infty$

$\lim\limits_{2^+} f_2(x) = \frac{3}{0^+} = +\infty$ $\Rightarrow$ A.V. $\equiv x = 2$

• $\lim\limits_{-2} f_2(x) = \text{"}\frac{3}{0}\text{"}$

$\lim\limits_{-2^-} f_2(x) = \frac{3}{0^+} = +\infty$

$\lim\limits_{-2^+} f_2(x) = \frac{3}{0^-} = -\infty$ $\Rightarrow$ A.V. $\equiv x = -2$.

Graphique a.

$f_3(x) = \frac{x^2 - 3x + 4}{x - 1}$ Dom $f_3 = \mathbb{R} \setminus \{1\}$

$= x - 2 + \frac{2}{x-1}$

A.O.

$$\begin{array}{rl|l} & x^2 - 3x + 4 & x - 1 \\ \hline - & (x^2 - x) & x - 2 \\ \hline & -2x + 4 & \\ - & (-2x + 2) & \\ \hline & 2 & \end{array}$$

$\Rightarrow$ A.O. $\equiv y = x - 2$.

A.V.

$\lim\limits_{1} f_3(x) = \frac{1-3+4}{0} = \text{"}\frac{2}{0}\text{"}$

$\lim\limits_{1^-} f_3(x) = \frac{2}{0^-} = -\infty$ $\Rightarrow$ A.V. $\equiv x = 1$

$\lim\limits_{1^+} f_3(x) = \frac{2}{0^+} = +\infty$

Graphique d.

$f_4(x) = \frac{x^3 - x^2 + 5x + 1}{x^2 + 1} = x - 1 + \frac{2 + 4x}{x^2+1}$

Dom $f_4 = \mathbb{R}$ $\Rightarrow$ Pas d'A.V.

A.O.

$$\begin{array}{rl|l} & x^3 - x^2 + 5x + 1 & x^2 + 1 \\ \hline - & (x^3 \qquad + x) & x - 1 \\ \hline & -x^2 + 4x + 1 & \\ - & (-x^2 \qquad - 1) & \\ \hline & 4x + 2 & \end{array}$$

Graphique c.