Contrôle formatif : Equations trigonométriques 20/03/20

Correctif.

1| ① $\sin x = \sin a$ $x = a + k.360°$ ou $x = 180° - a + k.360°$

② $\cos x = \cos a$ $x = \pm a + k.360°$

③ $\operatorname{tg} x = \operatorname{tg} a$ $x = a + k.180°$

2| ① $-\sqrt{3}\cos x + \sin x = +\sqrt{2}$

$\sqrt{3}\cos x - \sin x = -\sqrt{2}$

$r = 2$

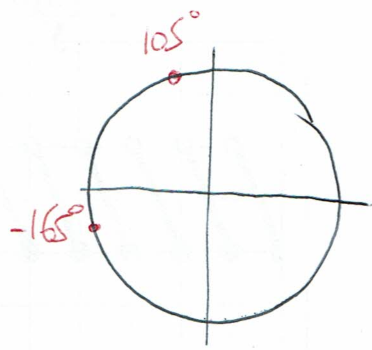
$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = -\frac{\sqrt{2}}{2}$

$\varphi = \operatorname{Arctg}\left(-\frac{1}{\sqrt{3}}\right) = -30°$

$\cos(x-(-30°)) = -\frac{\sqrt{2}}{2} = \cos(135°)$

$x + 30° = 135° + k.360°$ ou $x + 30° = -135° + k.360°$

$\boxed{x = 105° + k.360°}$ ou $\boxed{x = -165° + k.360°}$



② $-2\sin^2 x + 5\cos x - 1 = 0$

$-2(1-\cos^2 x) + 5\cos x - 1 = 0$

$2\cos^2 x + 5\cos x - 3 = 0$

$t = \cos x$

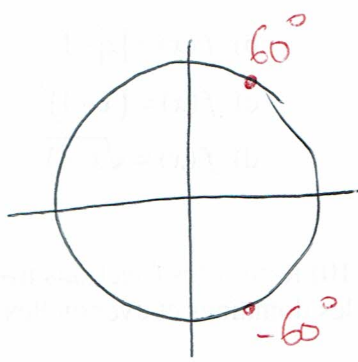
$2t^2 + 5t - 3 = 0$

$t_1 = \frac{1}{2}$ $t_2 = -3$

$\cos x = \frac{1}{2}$ $\cos x = -3$

impossible

$\boxed{x = \pm 60° + k.360°}$



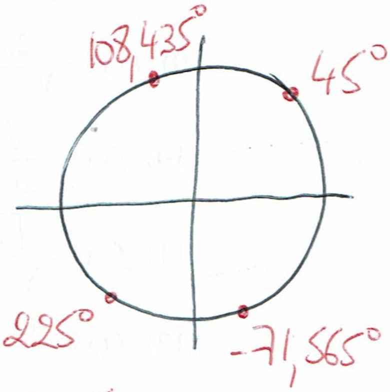
(3) $\sin^2 x + 2\sin x\cos x - 3\cos^2 x = 0$

Si $\cos x = 0$ alors $\sin^2 x + 0 - 0 = 0$

donc $\sin x = 0$ impossible.

donc $\cos x \neq 0$ et l'on peut diviser par $\cos^2 x$.

$tg^2 x + 2\,tg x - 3 = 0$



$t = tg x$

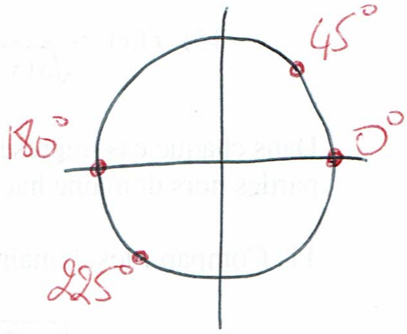
$t^2 + 2t - 3 = 0$

$t_1 = 1$ ou $t_2 = -3$

$tg x = 1$ ou $tg x = -3$

$\boxed{x = 45° + k.180°}$ ou $\boxed{x = -71{,}565° + k.180°}$

(4) $2\sin^2 x - \sin 2x = 0$



$2\sin^2 x - 2\sin x.\cos x = 0$

$2\sin x(\sin x - \cos x) = 0$

$\sin x = 0$ ou $\sin x = \cos x$

$\boxed{x = 0° + k.180°}$ $\quad \sin x = \sin(90° - x)$

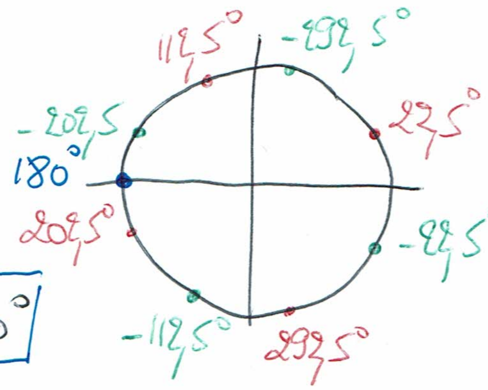
$x = 90° - x + k.360°$ ou $x = 180° - (90° - x) + k.360°$

$\boxed{x = 45° + k.180°}$ ou $0 = 90° + k.360°$

impossible

(5) $\cos 5x + \cos 3x + 2\cos 4x = 0$

$2\cos 4x.\cos x + 2\cos 4x = 0$



$2\cos 4x(\cos x + 1) = 0$

$\cos 4x = 0$ ou $\cos x = -1$

$4x = \pm 90° + k.360°$

$\boxed{x = \pm 22{,}5° + k.90°}$ ou $\boxed{x = 180° + k.360°}$

⑥ $\cos^4 x - \sin^4 x = 1$

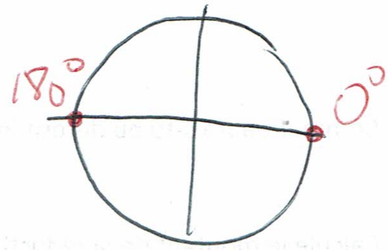
$$(\cos^2 x - \sin^2 x)\underbrace{(\cos^2 x + \sin^2 x)}_{=1} = 1 \qquad (a^2 - b^2 = (a-b)(a+b))$$

$$\cos^2 x - \sin^2 x = 1$$

$$\cos 2x = 1$$

$$2x = 0^\circ + k.360^\circ$$

$$\boxed{x = 0^\circ + k.180^\circ}$$



⑦ $\cos 2x + \sin 2x = \cos^2 x$.

$$\cancel{\cos^2 x} - \sin^2 x + 2\sin x \cdot \cos x = \cancel{\cos^2 x}$$

$$\sin x(-\sin x + 2\cos x) = 0$$

$\sin x = 0$ ou $-\sin x + 2\cos x = 0$

$\boxed{x = 0^\circ + k.180^\circ}$

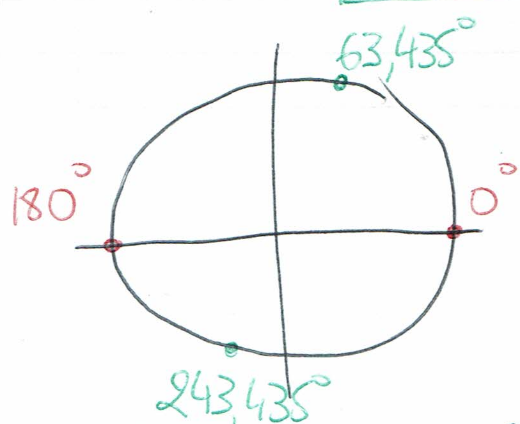
si $\cos x = 0$ alors $\sin x = 0$ impossible

donc $\cos x \neq 0$, on peut diviser par $\cos x$

$$\Rightarrow -\operatorname{tg} x + 2 = 0$$

$$\operatorname{tg} x = 2$$

$$\boxed{x = 63{,}435^\circ + k.180^\circ}$$



Autre méthode : $1 - 2\sin^2 x + 2\sin x \cos x = \cos^2 x$

$$\frac{1}{\cos^2 x} - 2\operatorname{tg}^2 x + 2\operatorname{tg} x = 1$$

$$1 + \operatorname{tg}^2 x - 2\operatorname{tg}^2 x + 2\operatorname{tg} x = 1$$

$$-\operatorname{tg}^2 x + 2\operatorname{tg} x = 0$$

$\operatorname{tg} x = 0$ ou $\operatorname{tg} x = 2$